

Overview of Small-World Networks and Application in Neuroimaging

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Small-World Phenomenon

- Q: How connected are two complete strangers?
- A: About six steps

Milgram's small-world experiment (1967)

Passing a letter from friends to friends



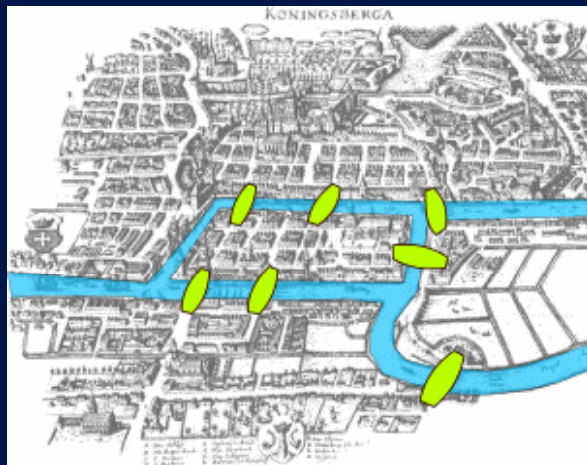
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Stanley Milgram

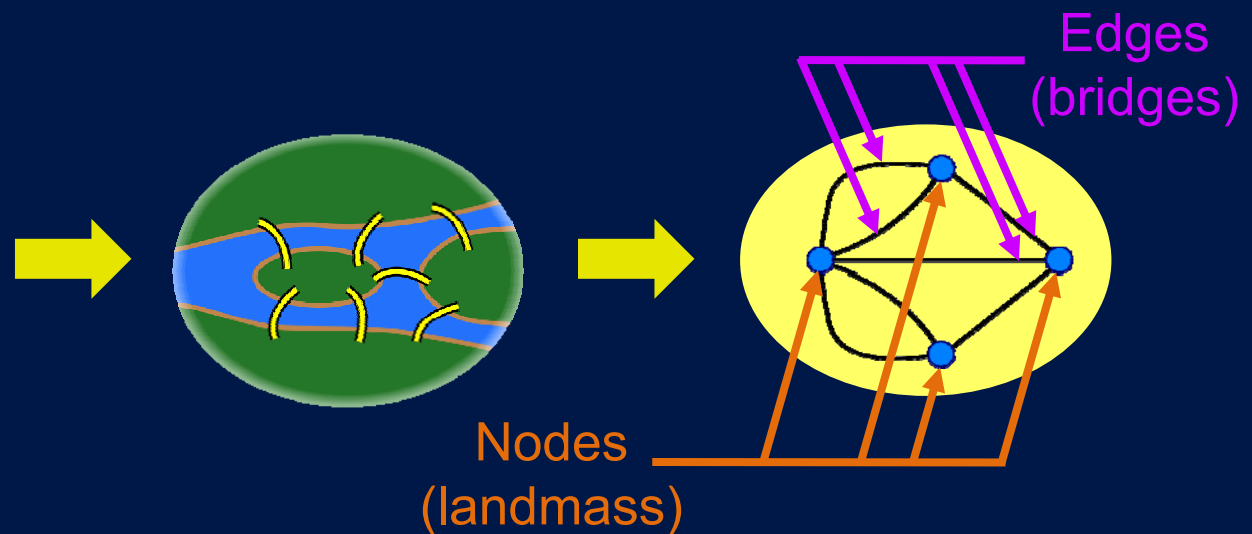


Graph Theory

- Graph: representation of relationships by
 - Nodes corresponding to each unit
 - Edges connecting nodes



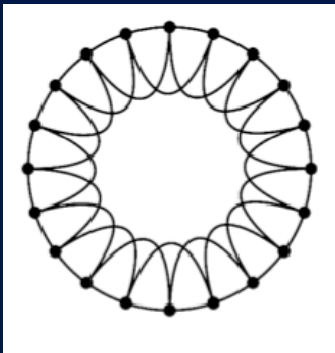
Bridges in Königsberg,
ca. 1736



Small-World Networks

- Small-world phenomenon meets graph theory (by Watts & Strogatz, Nature 1998)

Regular network

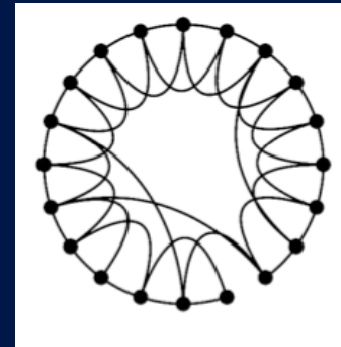


Sitting at a stadium

Re-writing
a few edges
randomly



Small-world network



Mobile phones at a stadium

Small-World Networks

- Small-world phenomenon meets graph theory (by Watts & Strogatz, Nature 1998)

Small-world network characteristics

Efficient long-distance communication

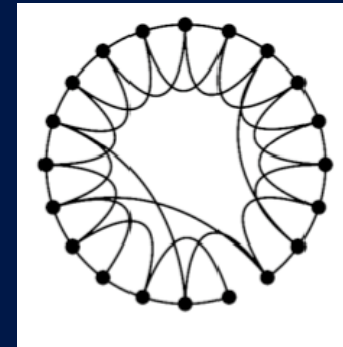
→ Any node is just a few steps away

Clustering

→ Highly interconnected neighborhoods

Global distribution & Local specialization

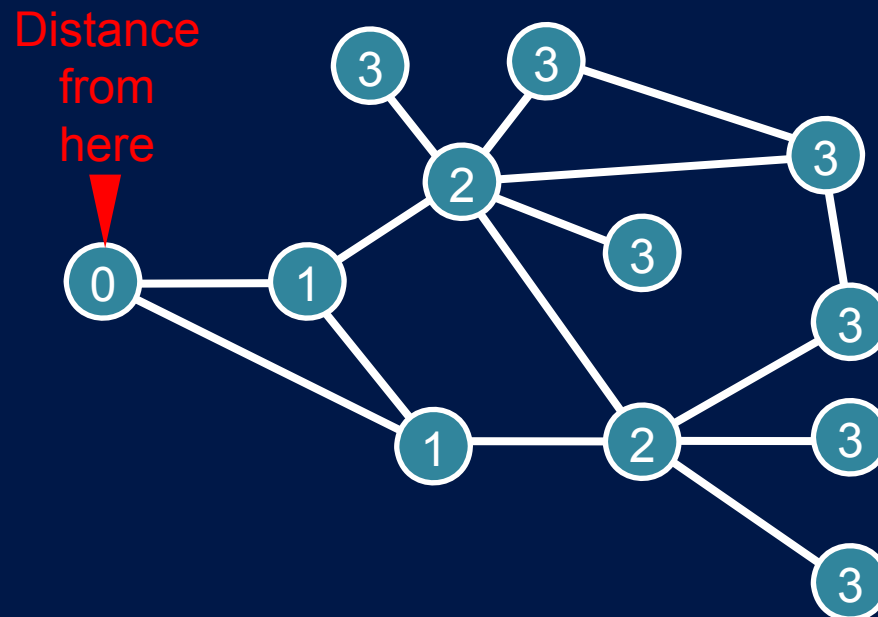
Small-world network



Mobile phones at a stadium

Small-World Characteristics

Efficient long-distance connection \rightarrow *path length L*
“L steps of separation” summarizing shortest distances between nodes

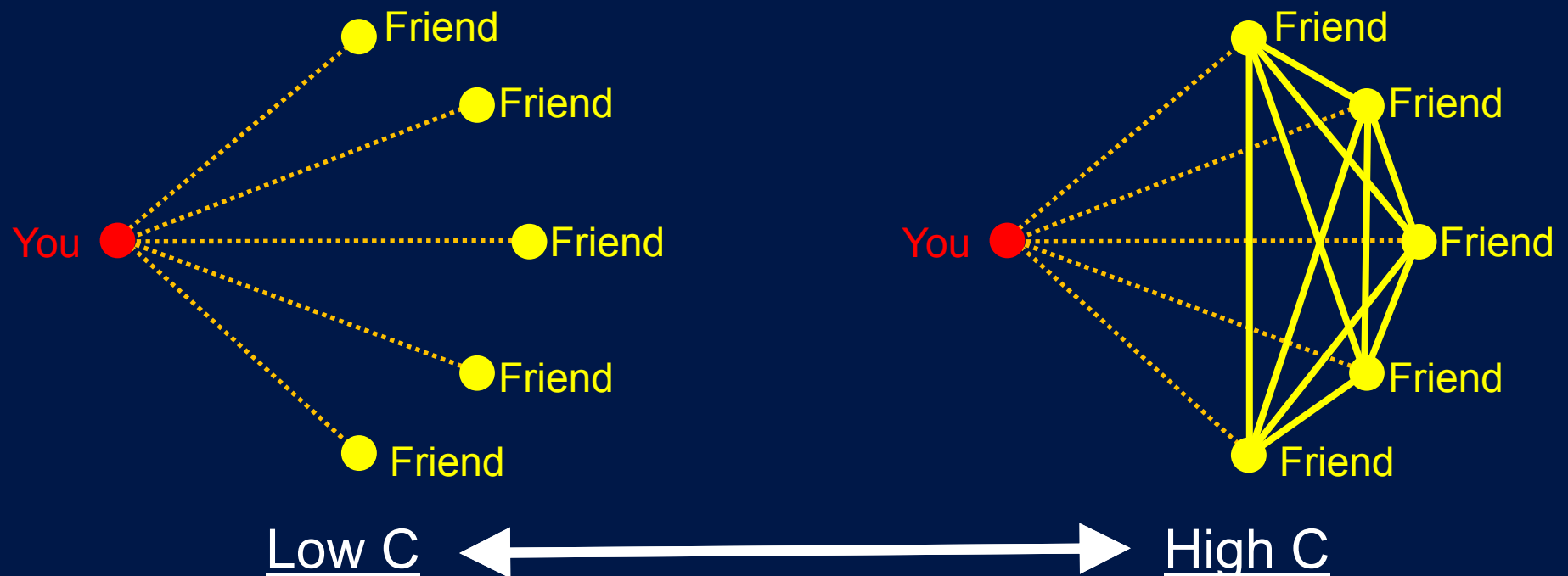


Small-World Characteristics

Tight local interconnection \rightarrow *clustering coefficient* C

Likelihood of nodes being interconnected

\rightarrow Probability of your friends being also friends to each other



Efficiency

- Alternative metrics to L & C
- Scaled from 0 to 1



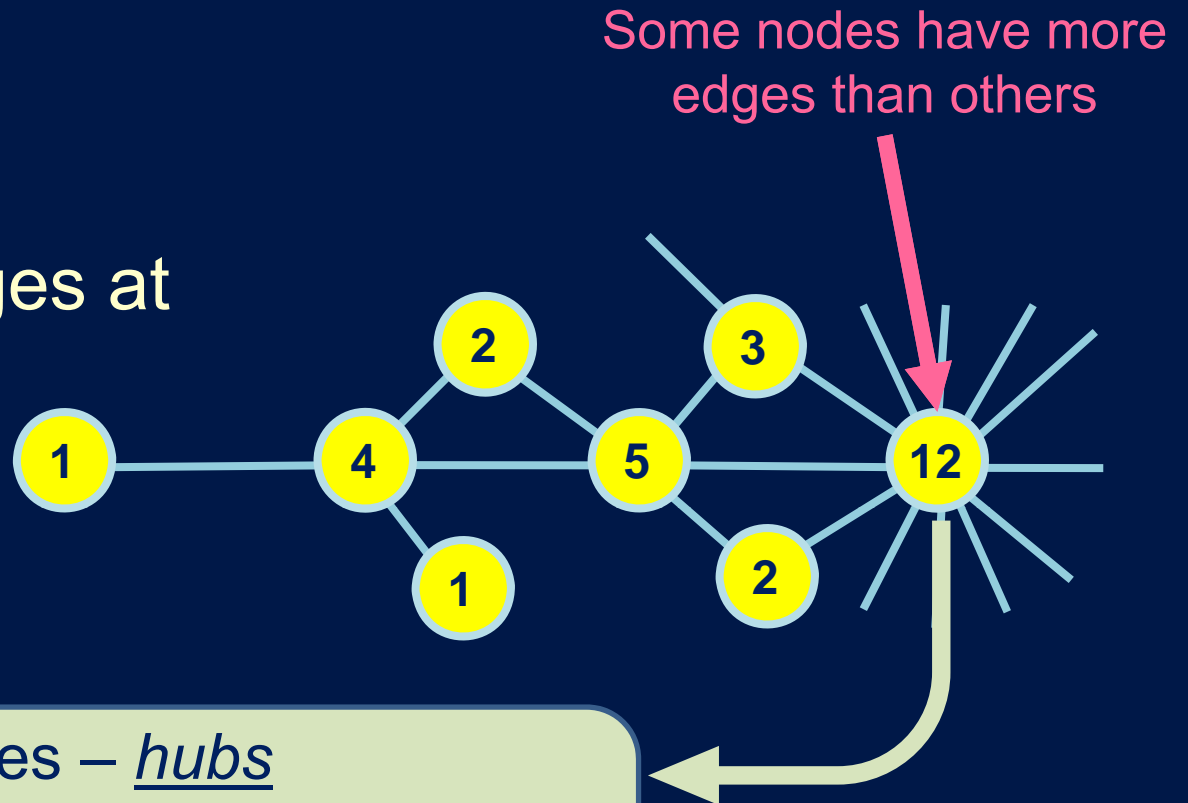
- Global efficiency (E_{glob}): for long-distance communication
- Local efficiency (E_{loc}): for tight local interconnection

Efficiency can be calculated

- For entire network
- At each node

Degree

- Number of edges at each node



High-degree nodes – hubs

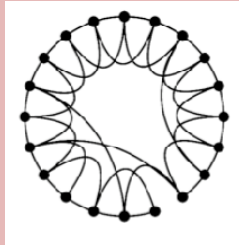
- Mediating many connections
- Vulnerability

Failure at hub → failure of entire network

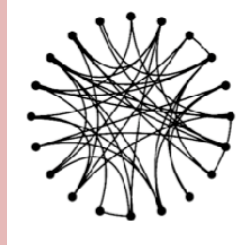
Small-Worldness σ

- Relative to a random network
 - Same number of nodes & edges

Small-world network



Random network



Clustering coefficient

C: very large

C_{rand} : small

Path length

L: very small

L_{rand} : very small

Small-world metric

$$\sigma = \frac{C/C_{rand}}{L/L_{rand}}$$

$\sigma > 1 \rightarrow$ small-world network

Humphries et al, Proc R Soc B (2006)

Examples

- Small-world networks are everywhere

Technological: power grid, internet, airline network

Social: Hollywood actors, Fortune 500 CEOs

Biological: yeast gene expression, nervous system

Entire neuronal network of *C. elegans*

Small-world network structure

→ Perhaps brain is a small-world network?

Anatomical Brain Networks

Animal cortical network

Histological tracing

→ Small-world network structure in

- *Macaque visual cortex* ($\sigma = 1.42$)
- *Macaque whole brain* ($\sigma = 2.63$)
- *Cat whole brain* ($\sigma = 1.67$)

Sporns & Zwi, Neuroinformatics (2004)

Human cortical connectivity

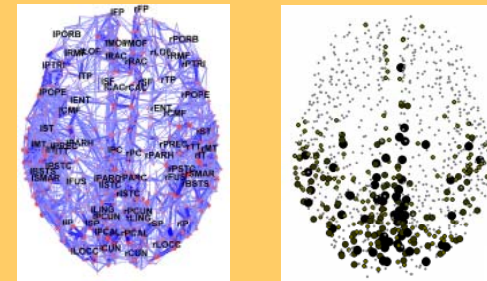
In-vivo structural imaging

→ Small-world network structure in

- *Cortical thickness network* ($\sigma = 2.04$)

He et al, Cereb Cortex (2007)

- *DSI-based connectivity network*
($\sigma = 10.64$)



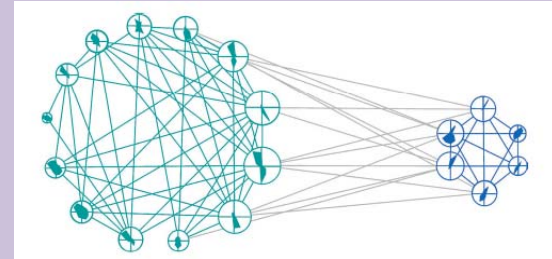
Hagmann et al., PLoS Biology (2008)

Functional Brain Networks

Neuronal level

Neuronal synchrony network in cat visual cortex ($\sigma = 1.79$)

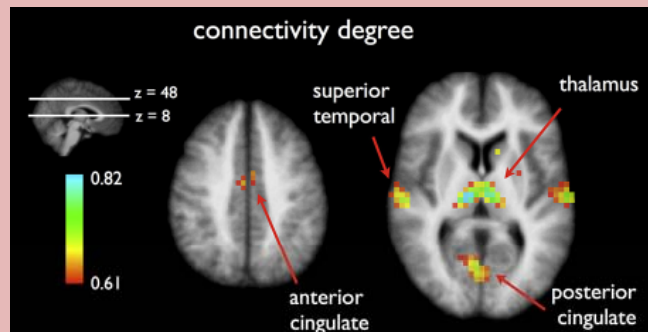
Yu et al., Cereb Cortex (2008)



Whole-brain functional connectivity networks

MEG during resting ($\sigma = 1.9$) & finger tapping ($\sigma = 1.7$) Bassett et al., PNAS (2006)

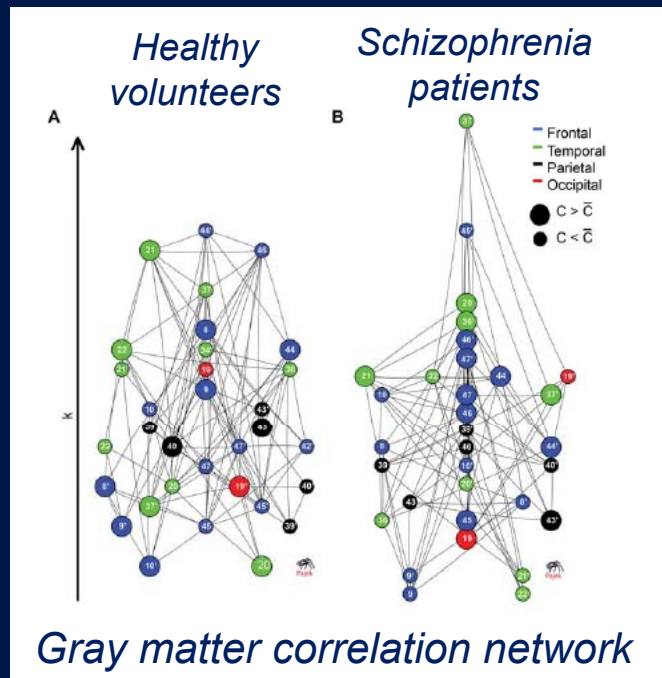
fMRI during resting ($\sigma > 30$)



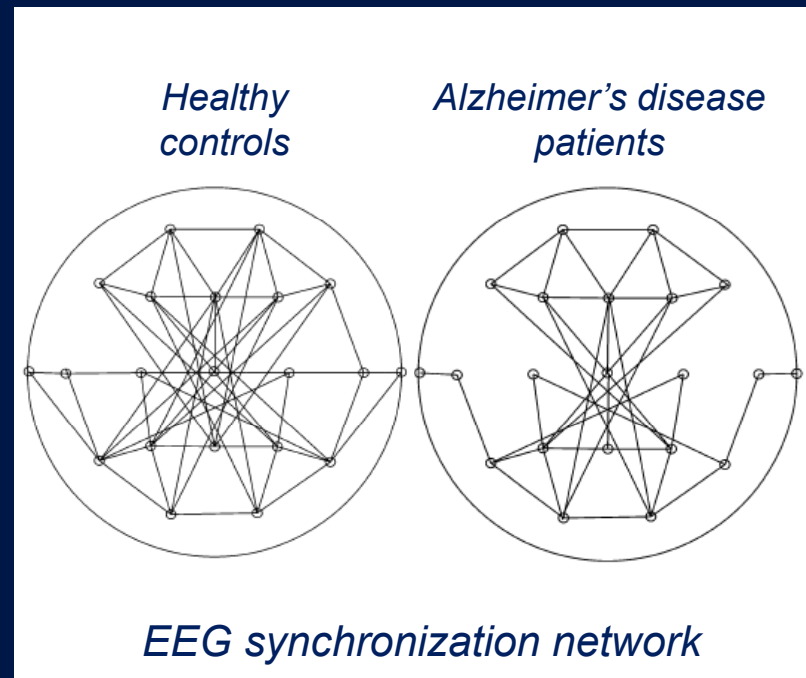
van den Heuvel et al., NeuroImage (2008)

Disease Brain Networks

- Network structure is altered
→ Neurological / psychiatric disorders



Bassett et al., J Neurosci (2008)



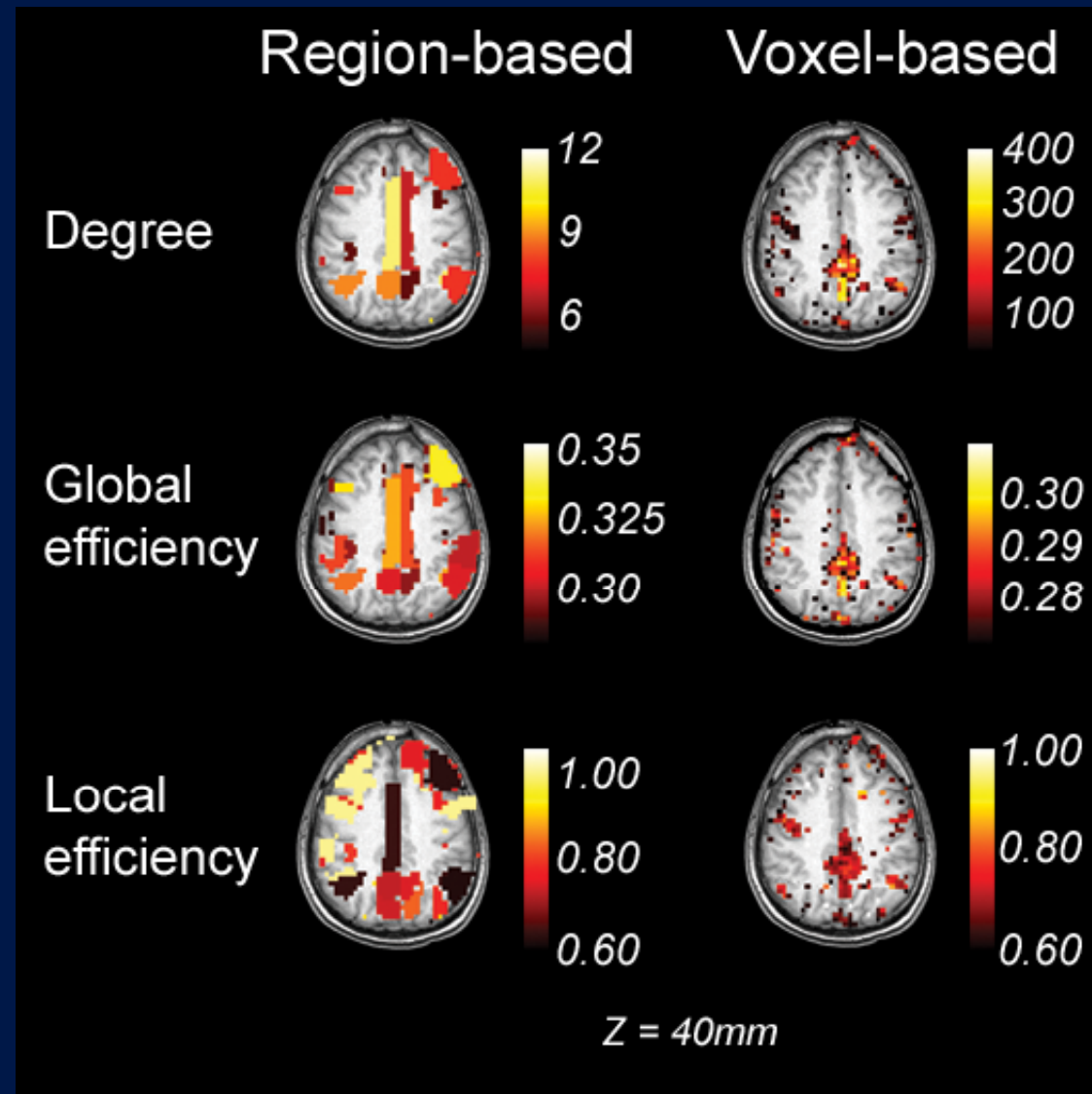
Stam et al., Cereb Cortex (2007)

Voxel-based Networks

(375-SU-AM)

Functional connectivity network

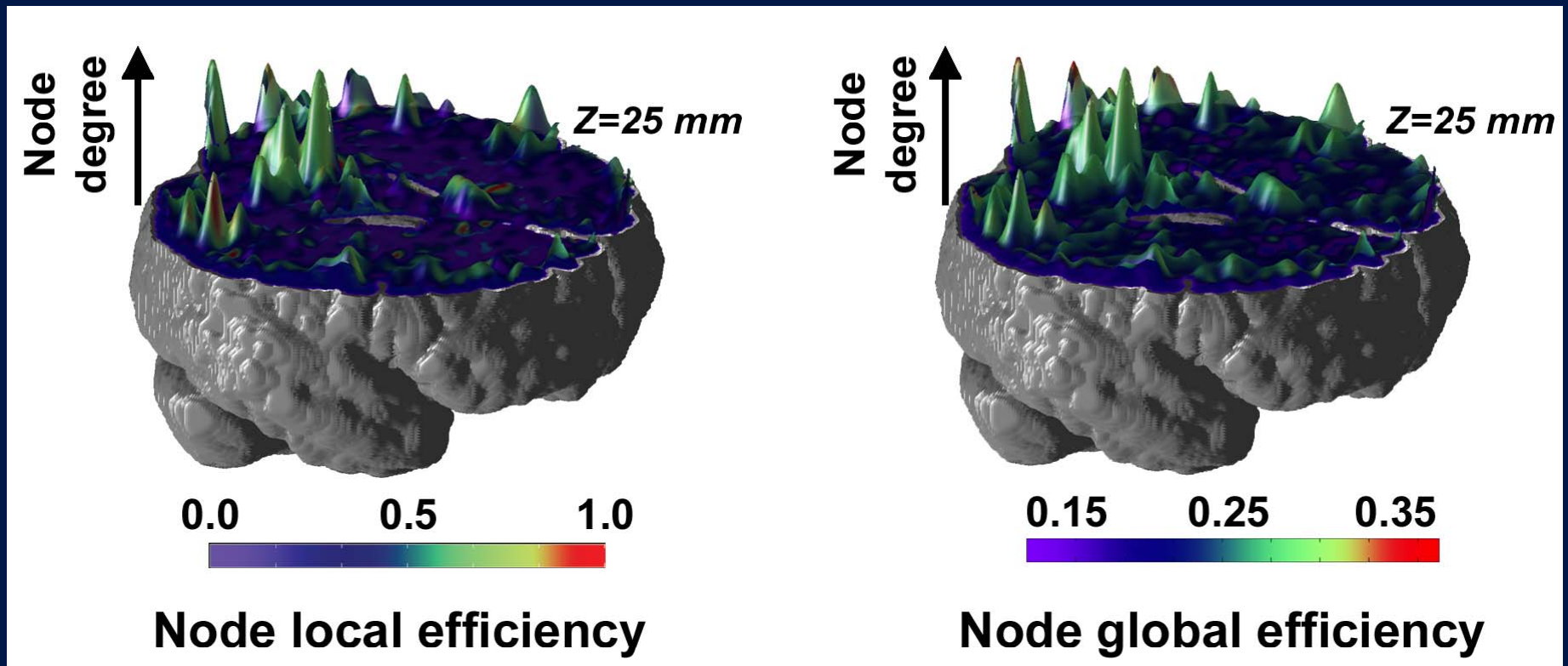
Resting-state fMRI data



Voxel-based Networks

(375-SU-AM)

- Node metric co-localization



Coming Up Next...



Olaf Sporns *Indiana University, USA*

Small-World Networks –
A Bridge from Structure to Function



Ed Bullmore *University of Cambridge, UK*

Fractal Scaling of Small-World Functional Networks



Danko Nikolić *Max Planck Institute, Germany*

Synchronization and Small-World Networks

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